Permutations & combinations

Multiplication Theorem
(Fundamental Principle of counting)

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If one event can be performed in 'm' ways & another event can be performed in 'n' ways

Then Total ways of Doing Two events one by one $= m \times n$

eg Maths Books $= m_1, m_2, m_3$

Accounts Books $= a_1, a_2$

one book of maths & one of Accounts can be selected in $3 \times 2 = 6$ ways

Factorial (!)

$n!$ = product of first 'n' natural numbers

eg $4! = 4 \times 3 \times 2 \times 1 = 24$

eg $8! = 8 \times 6 \times \cdots 3 \times 2 \times 1 = 40320$

$$\boxed{0! = 1}$$

$$n! = n(n-1)(n-2)(n-3) \cdots\cdots 3 \cdot 2 \cdot 1$$

$$(2n)! = 2n(2n-1)(2n-2) \cdots 3 \cdot 2 \cdot 1$$

$$(n+2)! = (n+2)(n+1)\,n(n-1) \cdots 3 \cdot 2 \cdot 1$$

$\boxed{1(1!) + 2(2!) + 3(3!) + \cdots + n(n!) = (n+1)! - 1}$

Eg $1(1!) + 2(2!) + 3(3!) = 4! - 1$

Permutations

Arrangement of elements where order of elements is important

Eg elements : 1, 2, 3

Permutations : 123, 132, 213
231, 312, 321

Total permutations of 'n' elements when 'r' elements are used at a time

$$= {}^{n}P_{r} = \frac{n!}{(n-r)!}$$

where $0 \le r \le n$

eg ${}^{8}P_{3} = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8 \times 7 \times 6 \times \cancel{5!}}{\cancel{5!}}$

$$\underbrace{{}^{8}P_{3}} = \underbrace{8 \times 7 \times 6}$$

3 factors

Eg $^{10}P_4 = \frac{10!}{6!} = \underbrace{10 \times 9 \times 8 \times 7}_{4 \text{ factors}}$

when some elements repeat with frequencies f_1, f_2 & f_3

Then Total Permutations $= \frac{n!}{f_1!\, f_2!\, f_3!}$

Eg ALLAHABAD

Total elements = 9

$A \rightarrow 4$, $L \rightarrow 2$

Total Permutations $= \frac{9!}{4!\, 2!}$

Circular Permutations

→ Total circular permutations of 'n' elements $= (n-1)!$

→ Total circular permutations when in each permutation have different neighbour or Necklace formation

$$= \frac{(n-1)!}{2}$$

→ Sum of all permutations when some digits are given

$$= \begin{bmatrix} \text{Sum of} \\ \text{given} \\ \text{Digits} \end{bmatrix} \times \frac{n!}{n} \times 11111 \ldots \text{n-times}$$

Q Digits : 1, 3, 5 & 8

Sum of all 4 digit numbers which can be made without repitition $= 17 \times \frac{4!}{4} \times 1111$

$= 1,13,322$

Combinations

Selection of elements where order of selection is not important.

Eg Elements : a, b, c
selection of 2 elements
ab, ac & bc

$${}^{n}C_{r} = \frac{{}^{n}P_{r}}{r!}$$

or

$${}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

Eg $^8C_2 = \frac{8!}{2!\,6!} = \frac{8\times 7}{2} = 28$

$^nC_0 = 1$

$^nC_n = 1$

$^nC_1 = n$

$^nC_2 = \frac{n(n-1)}{2}$

$^nC_r = {}^nC_{n-r}$

$^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$

If ${}^nC_a = {}^nC_b$

then $a = b$ or $a + b = n$

${}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = (2)^n$

${}^nC_1 + {}^nC_2 + {}^nC_3 + \cdots + {}^nC_n = (2)^n - 1$

Total handshakes in a group of n person $= {}^nC_2$

Total Diagonals in a polygon with n sides $= {}^nC_2 - n$

$= \frac{n(n-3)}{2}$

When 'n' non collinear Points are given

Then Total lines = ${}^{n}C_2$

Total Triangles = ${}^{n}C_3$

When 'n' points are given out of which 'm' points are collinear Then

Total lines = ${}^{n}C_2 - {}^{m}C_2 + 1$

Total triangles = ${}^{n}C_3 - {}^{m}C_3$